



The Basics of Numerical Filtering

Creative number crunching can smooth out data in a control loop.

To coffee drinkers and auto mechanics, "filtering" extracts unwanted contaminants from a stream of valuable material. Control engineers, on the other hand, use *numerical filters* to extract streams of valuable data from measurement noise and other extraneous information (see the "Filtering the feedback" graphic).

The "Running average" sidebar shows how the most basic numerical filter—a *running* or *moving average*—can extract the average value from a sequence of recent process variable measurements. Averaging is an especially effective method for determining the true value of a process variable when the measuring device is prone to unpredictable fluctuations commonly described as noise. Measurement *noise* affects all industrial instruments to one degree or another. Variations in ambient conditions (especially temperature)

or interference from nearby electrical sources can cause an unfiltered instrument to report a skewed reading. Sometimes the measurement technology itself is not entirely accurate, causing unfiltered measurements to deviate from the process variable's true value.

If deviations are brief, random, and equally distributed between positive and negative values, the noise will contribute no net effect to the average of recent measurements, so the readings reported by a filtered instrument will be accurate. Even if the noise is not strictly random and has a non-zero average or *mean*, an averaging filter can compensate by detecting the mean and subtracting it from the instrument's readings.

Alternative methods

The ability to extract accurate measurements from noisy data makes numerical filtering de

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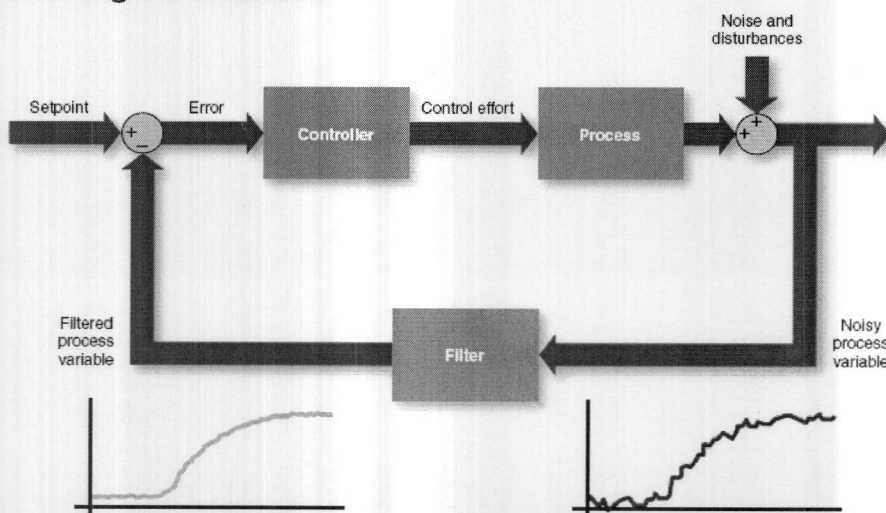
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Also read: **Multivariable Controllers Balance Competing Objectives**, from Sept., 2008 and

Analyzing Closed Loop Behavior with Convolution, from July, 2008 at www.controleng.com/archive.

Filtering the feedback



Source: Control Engineering

A numerical filter inserted in the feedback path of a control loop can reduce the noise that often corrupts the process variable measurements. Less noise in the feedback path typically leads to smoother control efforts.

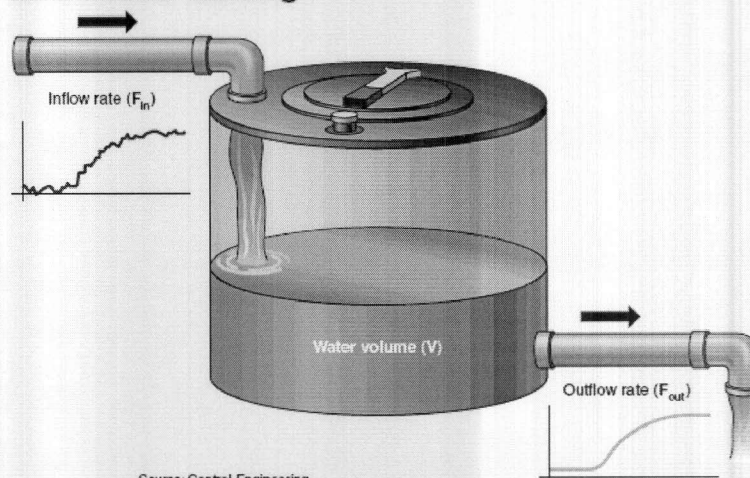
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rigueur for virtually all industrial instruments. However, most use more sophisticated and effective filtering techniques than a simple running average.

For example, a *weighted average* discounts older measurements in favor of newer ones when computing the instrument's reading, as in the "Weighted average" sidebar. Unlike a running average, this technique recognizes that the process variable changes over time so that recent measurements are likely a more accurate depiction of the current process variable than older measurements.

The *forgetting factor* technique is similar, though more computationally efficient. It too gives greater weight to more recent measurements but requires fewer calculations and less memory than either a moving or weighted average. See the "Forgetting factor" sidebar.

Mechanical filtering



At time $k\Delta t$, water flows out of this tank at a rate which is proportional to the volume of water it currently holds:

$$F_{out}(k) = A \cdot V(k)$$

The value of the constant A depends on the geometry of the tank. A tall, narrow tank would have a larger value of A than a short, wide tank since a taller tank generates more pressure behind the outflow when filled with the same volume of water. If the same relationship holds true a fraction of a second later at time $(k+1)\Delta t$, then

$$F_{out}(k+1) = A \cdot V(k+1)$$

The volume's rate of change over that interval is given by the difference between the inflow and outflow rates:

$$\frac{V(k+1) - V(k)}{\Delta t} = F_{in}(k) - F_{out}(k)$$

Combining these three relationships yields

$$F_{out}(k+1) = \lambda \cdot F_{out}(k) + (1 - \lambda) \cdot F_{in}(k)$$

where

$$\lambda = 1 - A \cdot \Delta t$$

Note that this relationship between the tank's inflow and outflow rates has the same form as a forgetting factor filter. That is, the tank averages the inflow rate to produce a smoother outflow rate, all by strictly mechanical means.

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Running average

The output of a running average filter at time $(k+1)\Delta t$ is the average of the last $n+1$ inputs sampled at intervals of Δt seconds:

$$F_{out}(k+1) = \frac{F_{in}(k) + F_{in}(k-1) + \dots + F_{in}(k-n)}{n+1}$$

For the purposes of instrumenting a feedback control loop, the filter's inputs would be the $n+1$ most recent process variable measurements, and the output would be the filtered reading reported by the instrument. Note that the output can also be computed *recursively* according to

$$F_{out}(k+1) = F_{out}(k) + \frac{F_{in}(k) - F_{in}(k-n-1)}{n+1}$$

This alternative approach involves fewer calculations but still requires storing all of the last $n+1$ inputs somewhere in the filter's memory so that the value of $F_{in}(k-n-1)$ can be recalled when needed.

Filtering without a computer

The forgetting factor technique also has the advantage of easy implementation with simple mechanical or electrical devices, and does not require a computer.

Consider, for example, the problem of extracting a smooth flow of water from an erratic source that is subject to random surges. A closed-loop flow controller could be installed with a valve and flow meter to counteract fluctuations in the upstream flow rate, but the same effect could be achieved by simply passing the stream through a tank as shown in the "Mechanical filtering" graphic.

If the tank is large enough, the water level inside will eventually stabilize at a

point where the outflow rate equals the average inflow rate. Fluctuations in the erratic inflow will be averaged out over time by the tank's storage capacity. The mathematical relationship between the two flow rates (as shown in the graphic) will have exactly the same form as a forgetting factor filter, though λ is typically described as a *time constant* when such a filter is implemented by physical, rather than strictly numerical means.

Challenges

This example also demonstrates one of the principal drawbacks of using a filter to eliminate noise from a data stream (or in this case, a literal stream). Filtering slows down the measuring process.

Weighted average

The output of a weighted average filter depends more heavily on the values of the more recent inputs:

$$F_{out}(k+1) = \frac{F_{in}(k) + w \cdot F_{in}(k-1) + \dots + w^n \cdot F_{in}(k-n)}{1 + w + \dots + w^n}$$

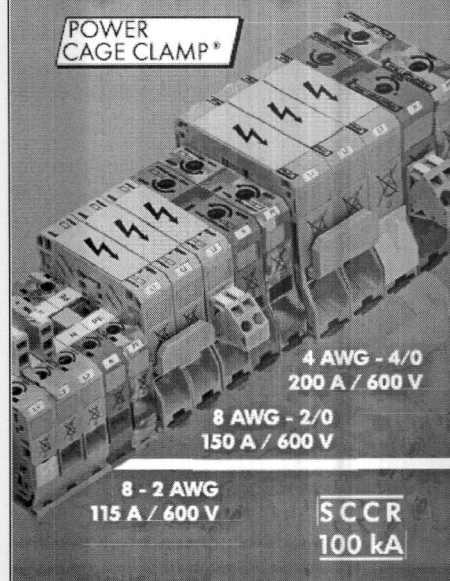
The weighting factor w can be assigned a value between 0 and 1 in order to quantify the relative importance of older vs. newer inputs. If $w=0$, the older inputs are discounted entirely. If $w=1$, all inputs are given equal weight just like a running average. And like a running average, the output of a weighted average filter can be computed more efficiently with a recursive formula:

$$F_{out}(k+1) = w \cdot F_{out}(k) + \frac{F_{in}(k) - w^n \cdot F_{in}(k-n-1)}{1 + w + \dots + w^n}$$

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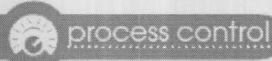
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Imagine that the supply has been shut off somewhere upstream. Until the tank drains the outflow continues and does not indicate that the inflow rate has dropped to zero. A large tank would take longer to drain, prolonging the outflow and inflow disparity. In other words, the tank would take a long time to "forget" the old inflow rate, so its forgetting factor/time constant would be particularly large. A large tank's storage capacity can average inflow rate fluctuations over a longer period of time, minimizing the effect of any individual surge. Better averaging is also characteristic of a numerical filter with a large time constant.

Process designers applying this mechanical filtering technique would size the tank to fit their objectives. A larger tank with a larger time constant would produce better averaging, but a smaller tank with a smaller time constant would be more responsive.

The same design trade-offs affect

strictly numerical filters as well, especially those in feedback control loops. A filter with a long time constant does a better job of eliminating measurement noise, but it averages actual changes in the process variable over time, so it takes longer for the controller to see the effects of its previous control efforts. If the controller is impatient, it could

apply more correction than necessary, much as when the process itself is slow to respond because of excessive dead-time. (See "Dealing with Deadtime", *Control Engineering*, July 2005.)

Sophisticated filtering

These simple averages are only the most basic filtering techniques. *Nth order filters* have *N* forgetting factors or time constants for improved performance. *High pass filters* preserve the high frequency components of the data stream rather than the low frequency components. *Band pass filters* discard both the highest and lowest frequency components and leave the mid-frequency components more or less intact. *Band stop filters* perform the converse function.

All these techniques have advantages and disadvantages, and all involve complex mathematical analysis. Fortunately, *signal theory*—the study of data streams and methods for filtering them—is one of the most mature facets of control engineering since these techniques were developed in electrical engineering and telecommunications long before Ziegler and Nichols tuned their first loop.

As a result, there is a host of design tools and software packages to assist developing a filter with desired characteristics. These tools can also be used to design entire control loops since a feedback controller is essentially a filter that acts on the measured error between the process variable and the setpoint to produce a sequence of control efforts with desired characteristics. **ce**

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Forgetting factor

A filter equipped with a forgetting factor λ between 0 and 1 gives more weight to more recent inputs much like a weighted average filter:

$$F_{out}(k+1) = (1-\lambda) \cdot F_{in}(k) + \lambda \cdot (1-\lambda) \cdot F_{in}(k-1) + \lambda^2 \cdot (1-\lambda) \cdot F_{in}(k-2) + \dots$$

However, this technique incorporates *all* past inputs into the filter's output, not just the last $n+1$. The input measurement that is $m\Delta t$ seconds old is discounted by a factor of λ^m , so the filter essentially "forgets" older inputs at a rate given by λ . If $\lambda=0$, the older inputs are forgotten immediately. If λ is close to 1, the older inputs are forgotten relatively slowly. Note that the recursive version of the forgetting factor formula is the most computationally efficient of all averaging filters:

$$F_{out}(k+1) = \lambda \cdot F_{out}(k) + (1-\lambda) \cdot F_{in}(k)$$

Only the current input and the current output need be stored at time $k\Delta t$ for use at time $(k+1)\Delta t$.

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